Credit Default Swap Pricing
based on ISDA Standard Upfront Model

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In late 2010, China unveiled its onshore credit derivatives market with the launch of two Credit Risk Mitigation (CRM) products: the CRM Agreement (CRMA) and the CRM Warrant (CRMW).

However, the market have remained languid since the launch more than five years ago.

In the wake of the disappointing performance of first generation of credit derivative instruments, in mid 2016, the National Association of Financial Markets Institutional Investors (NAFMII) ratified a major revision of its credit derivatives guidelines with the introduction of two new products: CDS and CLN, which return to an international CDS standard feature.

In 31st Oct 2016, Some Banks (Bank Of China, China Construction Bank, etc.) start their first trade in CDS with total notional 300MM CNY.

This Presentation focus on: Standard CDS Contract Pricing based on ISDA Standard Upfront Model
For $i$-th period,

- **Premium Payment** = $N \times DCC(s_i, e_i) \times C$ (no default)
  
  = $N \times DCC(s_i, \tau) \times C$ (default at time $\tau$)

- **Default Payment** = $N \times (1 - RR)$

where:

- $s_i$: accural start date
- $e_i$: accural end date
- $\tau$: default date
- $C$: fixed coupon amount
- $N$: Notional Amount
- $DCC(t_1, t_2)$: Day Count Convention between date $t_1$ and $t_2$
- $RR$: Recovery Rate
Differences before and after ‘Big Bang’

- ‘Big Bang’: in 2009 ISDA (International Swaps and Derivatives Association) issued the ‘Big Bang’ protocol in an attempt to restart the market by standardising CDS contracts.

- Before ‘Big Bang’: CDSs were traded at par (have zero cost of entry). CDSs were quoted by *par spread*, which makes the *clean PV* of the CDS zero. For all protection buyers, the cost of entry is zero and they only needs to pay regular coupon payments.

- After ‘Big Bang’: All CDS contracts now have a *standard coupon and up-front charge*, quoted as a percentage of the notional - *Points Up-Front (PUF)* or *par spread*. This amount is quoted as if it is paid by the protection buyer. In other words, protection buyer will pay or receive an upfront charge when they enter into a CDS contract.
However, *Upfront Amount* can also be negative, in which case it is paid to the protection buyer.
The Standard CDS Contract

- **Legal Effective Date.** This was $T + 1$ (i.e. the same as the step-in date), which could cause risk to offsetting trades (as there may be a delay in credit event becoming known). It is now $T - 60$ (credit events) or $T - 90$ (succession events).

- **Accrued Interest.** For legacy trades only accrued interest from the step-in date ($T + 1$). If the trade date was more than 30 days before the first coupon date, the first coupon is reduced to just be the accrued interest over the shortened period (short stub); if there are less than 30 days, nothing is paid on the (nominal) first coupon and on the second coupon (effectively the first) the full accrued over the extended period (long stub) is paid - i.e. the normal premium for this period plus the portion not paid on the first coupon date. Following this front stub period, normal coupons are paid. For standard CDS, interest is accrued from the previous IMM date (the prior coupon date), so all coupons are paid in full. This accrued interest (from the prior coupon date to the step-in date) is rebated to the protection buyer, on the cash settlement date.

- **Coupons.** Previously CDSs were issued with zero cost of entry, so the coupon was such that the fair value of the CDS was zero (par spread). Now CDSs are issued with standard coupons, and and upfront free is paid.
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Introduction to ISDA Model

The ISDA Standard CDS Model is maintained by Markit. It is written in the **C language** and is the evolution of the JP Morgan CDS pricing routines. The source code is written by JP Morgan Engineer.
Introduction to ISDA Model

Their documentations are mainly focused on standard contract specification and we can only get clues of mathematical formula form its original source code. However, the source code have **large amount of codes** and is hard to read.

Luckily, OpenGamma has summarized these source code based on their understanding. Thus, the following summary of ISDA model are mainly based on OpenGamma’s quant research paper and some textbooks.
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We will denote the year-fraction, $\triangle t$, between two dates, $t_1$ and $t_2$ as:

$$\triangle t = DCC(t_1, t_2)$$

ISDA Model:

- The DCC for premium payments: $ACT/360$
- The DCC for curves: $ACT/365F$
Interest Rate Curve

The Price of zero-coupon bond at some time $t$, for expiry at $T \geq t$, is defined as $P(t, T)$.

If the instantaneous **spot rate** $r(t)$ is deterministic, the price of a zero-coupon bond is given by

$$ P(t, T) = e^{-\int_t^T r(s)ds} \tag{1} $$

If $r(t)$ follows some stochastic process, the price of a zero-coupon bond is given by

$$ P(t, T) = \mathbb{E}^Q[e^{-\int_t^T r(s)ds} | \mathcal{F}_t] \tag{2} $$

Also, we can define the **instantaneous forward rates**, $f(t, s)$, $s \geq t$ and $f(t, t) = r(t)$, such that

$$ P(t, T) = e^{-\int_t^T f(t, s)ds} \iff f(t, s) = -\frac{\partial \ln[P(t, s)]}{\partial s} = -\frac{1}{P(t, s)} \frac{\partial P(t, s)}{\partial s} \tag{3} $$

Finally, we define the **continuously compounded yield** on the zero coupon bond $R(t, T)$, as

$$ P(t, T) = e^{-R(t, T)(T-t)} \iff R(t, T) = -\frac{1}{T-t} \ln[P(t, T)] \tag{4} $$

When $t = 0$ we may drop the first argument and simply write $P(T)$, $f(T)$ and $R(T)$ for the discount factor, forward rate and zero rate to time $T$. 
Credit Curve (Survival Probability Curve)

If we assume default is a Poisson process, with an deterministic intensity $\lambda(t)$. If the default time is $\tau$, then the probability of default over an infinitesimal time period $dt$, given no default to time $t$ is

$$
P(t < \tau \leq t + dt | \tau > t) = \lambda(t)dt \tag{5}
$$

The probability of surviving to at least time, $T > t$, (assuming no default has occurred up to time $t$) is given by

$$
Q(t, T) = P(\tau > T | \tau > t) = E(\mathbb{I}_{\tau > T} | \mathcal{F}_t) = e^{-\int_t^T \lambda(s)ds} \tag{6}
$$

If we assume intensity follows a stochastic process, then the survival probability is given by

$$
Q(t, T) = E^Q[e^{-\int_t^T \lambda(s)ds} | \mathcal{F}_t] \tag{7}
$$

Also, we can extend this analogy and define the forward hazard rate, $h(t, T)$ as

$$
Q(t, T) = e^{-\int_t^T h(t, s)ds} \iff h(t, s) = -\frac{\partial \ln[Q(t, s)]}{\partial s} = -\frac{1}{Q(t, s)} \frac{\partial Q(t, s)}{\partial s} \tag{8}
$$
and the zero hazard rate, $\Lambda(t, T)$ as

$$Q(t, T) = e^{-\Lambda(t, T)(T-t)} \iff \Lambda(t, T) = -\frac{1}{T-t} \ln[Q(t, T)]$$ (9)

The forward hazard rate represents the (infinitesimal) probability of a default between times $T$ and $T + dT$, conditional on survival to time $T$, as seen from time $t < T$. The unconditional probability of default between times $T$ and $T + dt$ is given by

$$\mathbb{P}(T < \tau \leq T + dt | \tau > t) = Q(t, T) h(t, T) = -\frac{\partial Q(t, T)}{\partial T}$$ (10)

When $t = 0$ we may drop the first argument and simply write $Q(T), h(T)$ and $\Lambda(T)$ for the survival probability, forward hazard rate and zero hazard rate to time $T$.

Note that when $t = 0$, equation is just the probability density function for default time $\tau$. 

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We assume that a continuous time interest rate and credit curve is extraneously given, and are defined from the trade date \((t = 0)\) to at least the expiry of the CDS.

We consider the prices at cash-settlement date rather than the trade date. Also, we take the trade date as \(t = 0\) and the maturity as \(T\). The step-in(effective protection date) is \(t_e\) and the valuation(cash-settle date) is \(t_v\).
The Protection Leg

The protection leg of a CDS only consists of a random payment of \( N(1 - RR(\tau)) \) at default time \( \tau \), if default time \( \tau \) is before the expiry of the CDS (time \( T \)). Thus, the present value of this leg is given by:

\[
PVP_{Protection\ Leg} = N \mathbb{E}^Q \left[ e^{-\int_{t_v}^{\tau} r(s)ds} (1 - RR(\tau)) \mathbb{I}_{\tau < T} \right]
= N(1 - RR) \mathbb{E}^Q \left[ e^{-\int_{t_v}^{\tau} r(s)ds} \mathbb{I}_{\tau < T} \right]
= - \frac{N(1 - RR)}{P(t_v)} \int_0^T P(s) \frac{dQ(s)}{ds} ds
= - \frac{N(1 - RR)}{P(t_v)} \int_0^T P(s)dQ(s)
\]

(11)

Where \( RR = \mathbb{E}^Q[RR(\tau)] \) is the expected recovery rate; second equality is given under assumptions that recovery rates are independent of interest, hazard rates and the default time; third equality is given under assumptions that interest rates and hazard rates are independent; \( \mathbb{E}^Q[\cdot] \) is the expectation taken under risk-neutral measure.
The Premium Leg

The premium leg consists of two parts: Coupon payments up to the expiry of the CDS, which cease if a default occurs; and a single payment of accrued premium in the event of a default (this is not included in all CDS contracts but is for SNAC). The Chinamoney’s summary of Chinese CDS specifications indicates that the Chinese version of CDS has accrued interest.

Assume there are $M$ remaining payments with

- payment times $t_1, t_2, \ldots, t_M$
- period end times $e_1, e_2, \ldots, e_M$
- year fraction $\triangle_1, \triangle_2, \ldots, \triangle_M$

The present value of the premiums only is

$$PV_{\text{premium only}} = NC E^Q \left[ \sum_{i=1}^{M} \triangle_i e^{- \int_{t_i}^{t_v} r(s) ds} I_{e_i < \tau} \right]$$

$$= \frac{NC}{P(t_v)} \sum_{i=1}^{M} \triangle_i P(t_i) Q(e_i) \quad (12)$$

The quantity $P(T)Q(T) \equiv B(T)$ is known as the risky discount factor - the PV of the premium payments is just the risky discounted value of the cash flows.
The Premium Leg

The second part of the premium leg is the accrued interest paid on default. If the accrual start and end times are $(s_1, e_1), \ldots, (s_i, e_i), \ldots, (s_M, e_M)$, its PV is given by

\[
P_{\text{accured interest}} = NC \mathbb{E}^Q \left[ \sum_{i=1}^{M} DCC(s_i, \tau) e^{-\int_{t_v}^{\tau} r(s) ds} \mathbb{I}_{s_i < \tau < e_i} \right]
\]

\[
= - \frac{NC}{P(t_v)} \sum_{i=1}^{M} \left[ \int_{s_i}^{e_i} DCC(s_i, t) P(t) \frac{dQ(t)}{dt} dt \right]
\]

\[
= - \frac{NC}{P(t_v)} \sum_{i=1}^{M} \left[ \eta_i \int_{s_i}^{e_i} (t - s_i) P(t) \frac{dQ(t)}{dt} dt \right] \quad (13)
\]

Where the third equality is due to different day count convention applied for curves and accrued interest. We usually use ACT/365 for measure year fractions for the curves and ACT/360 for accrued interest. In this case, we have $\eta_i = DCC_{\text{accured}}(s_i, e_i)/DCC_{\text{curve}}(s_i, e_i)$. 

Thus, the full PV of the premium leg is given by:

\[
P V_{\text{premium}} = P V_{\text{premium only}} + P V_{\text{accured interest}}
\]

\[
= \frac{N C}{P(t_v)} \sum_{i=1}^{M} \left[ \triangle_i P(t_i) Q(e_i) - \eta_i \int_{s_i}^{e_i} (t - s_i) P(t) \frac{dQ(t)}{dt} dt \right]
\]

(14)

This scales linearly with coupon \( C \). The Risky \( PV01 \) (RPV01) is defined as the value of the premium leg per unit of coupon, so:

\[
P V_{\text{premium}} = C \times \text{RPV01}_{\text{dirty}}
\]

(15)
Clean PV v.s. Dirty PV

The valuation we discussed above is the dirty PV, since it considers the accrued interest. For clarity we label the Risky PV01 given above, $RPV_{01_{dirty}}$ and RPV01 without a subscript as clean $RPV_{01}$. Thus, the cash settlement for the buyer of protection is given by:

$$PV_{dirty} = PV_{Protection\ Leg} - C \times RPV_{01_{dirty}}$$ \hspace{1cm} (16)

As with bonds, this value has a sawtooth pattern against time, driven by the discrete premium payments. To smooth out this pattern, the accrued interest between the accrued start date (immediately before the step-in date) and the step-in date (protection effective date T+1 business day adjusted) is subtracted from the premium payments. So

$$RPV_{01} = RPV_{01_{dirty}} - N \times DCC(s_1, t_e)$$ \hspace{1cm} (17)

Thus, we have

$$PV_{clean} = PV_{Protection\ Leg} - C \times RPV_{01}$$ \hspace{1cm} (18)

$$PV_{dirty} = PV_{Protection\ Leg} - C \times RPV_{01} - NC \times DCC(s_1, t_e)$$ \hspace{1cm} (19)
Points Up-front (PUF) and Par Spread

- **Points Up-front (PUF)**

\[
PUF = \frac{PV_{clean}}{N} \tag{20}
\]

- **Par Spread**: the coupon that makes the clean PV of the CDS zero is known as the par spread, \( S_p \), which is given by

\[
S_p(T) = \frac{PV_{Protection\ Leg}}{RPV01}
\]

\[
= - (1 - RR) \int_0^T P(s) \frac{dQ(s)}{ds} ds
\]

\[
= \sum_{i=1}^M \left[ \Delta_i P(t_i) Q(e_i) - \eta_i \int_{s_i}^{e_i} (t - s_i) P(t) \frac{dQ(t)}{dt} dt \right] - P(t_v) \times DCC(s_i, t_e) \tag{21}
\]
Points Up-front (PUF) and Par Spread

This trade follows new ISDA contract roll frequency: doc and website


Deal
REF Entity: International Business Machines Corp
Debt Type: Senior
REF Obligation: US459200HM60

Trade Date: 03/07/17
1st Accr Start: 12/20/16
1st Coupon: 03/20/17
Pen Coupon: 09/20/21

Maturity: 5Y: 12/20/21
Use curve recovery rate: True
Recovery Rate: 0.40

Calculator
Cash Settled On: 03/10/17
Cash Calculated On: 03/10/17
EDD: No
Price: 102,773,840
Principal: -277,384
Accrued (78 Days): -21,667
Cash Amount: -299,051

ISDA Standard Upfront Model (I)*
Valuation Date: 03/07/17
Spread DV01: 4,670.40
IR DV01: 67.79
Rec Risk (%): 71.53
Def Exposure: 6,277,384

Recovery Rate: 0.40
Term Pts Upf: Spread Prob
12/20/21: -2.77384086 39.2072 0.0312

View Term Structure

Bloomer: 320 BI 15:40 Lindt Key Annual Earnings Metrics: Analyzer
319 APW 15:40 Philippines-China/ Joint Commission Meeting
318 NS6 15:40 Berita Harian: Penganggur tak mengaku salahrogel teman wanita

**This application is based on the ISDA Std Model v1, developed and supported in collaboration with Markit Group Ltd.**

Summarized by Wu Chen (RMI)
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The ISDA Model makes the assumption that both yield curve and credit curve are piecewise constant in their respective (instantaneous) forward rates, which will simplify the calculation for the integrals in protection leg and premium leg.

- **Yield Curve**: Constructed from money market rates (spot (L)ibor rates) with maturities out to 1Y, and swap rates with maturities out to 30Y [Single Curve Discounting for OTC market, OIS Discounting for trades on exchange]
- **Credit Curve**: Calibrated from market price
We assume that the yield curve has \( n_y \) nodes at time \( \mathcal{T}^y = \{ t_1^y, t_2^y, ..., t_{n_y}^y \} \) and the credit curve has \( n_c \) nodes at \( \mathcal{T}^c = \{ t_1^c, t_2^c, ..., t_{n_c}^c \} \). At the \( i^{th} \) node of the yield curve the discount factor is given by \( P_i = \exp(-t_i^y R_i) \) and at \( i^{th} \) node of the credit curve the survival probability is given by \( Q_i = \exp(-t_i^c \Lambda_i) \). Then we have:

\[
Q(t) = \exp\left(- \left[ \frac{t_i^c \Lambda_i (t_{i+1}^c - t) + t_{i+1}^c \Lambda_{i+1} (t - t_i^c)}{\Delta t_i^c} \right] \right)
\]

for \( t_i < t < t_{i+1} \) \( (22) \)

where \( \Delta t_i^c = t_{i+1}^c - t_i^c \). The interpolation is linear in the log of the survival probability. Thus we have the forward hazard rate:

\[
Q(t) = \frac{-t_i^c \Lambda_i + t_{i+1}^c \Lambda_{i+1}}{\Delta t_i^c} \equiv h_{i+1}
\]

for \( t_i < t < t_{i+1} \) \( (23) \)

which justifies our previous statement that the forward rates are piecewise constant. This allows us to write

\[
Q(t) = Q_i e^{h_{i+1}(t-t_i^c)}
\]

\( (24) \)

\[
\Lambda(t) = \frac{\Lambda_i t_i^c + h_{i+1} (t - t_i^c)}{t}
\]

for \( t_i < t < t_{i+1} \) \( (25) \)
For times before the first node we have simply

$$Q(t) = e^{-h_1 t} \text{ for } 0 < t < t_1$$  \hspace{1cm} (26)

and after the last node

$$Q(t) = Q_{n_c} e^{-h_{n_c} (t-t_{n_c}^c)} \text{ for } t > t_{n_c}^c$$  \hspace{1cm} (27)

This property also holds for yield curve constructed from the money market rates and swap rates. In summary, the both forward interest rate curve and forward hazard rare curve are *piecewise constant curves*.

We let the chronologically ordered combined set of unique nodes be

$$\mathcal{T} = \mathcal{T}^y \cup \mathcal{T}^c = \{t_1, t_2, ..., t_n\}$$

where $n \leq n_y + n_c$. Obviously, we are able to guarantee that both the forward interest rate and forward hazard rate are constant.
Summary: Piecewise constant and flat extrapolation for (instantaneous) forward rate / forward hazard rate.
We use following notation for later derivation of formulas.

- $Q_i \equiv Q(t_i)$, $P_i \equiv P(t_i)$
- $\Delta t_i = t_{i+1} - t_i$
- $f_i$: the constant forward interest rate when $t_{i-1} < t < t_i$
- $h_i$: the constant forward hazard rate when $t_{i-1} < t < t_i$
- $\hat{f}_i \equiv f_i \Delta t_{i-1} = \ln(\overline{P}_{i-1}) - \ln(\overline{P}_i)$
- $\hat{h}_i \equiv h_i \Delta t_{i-1} = \ln(\overline{Q}_{i-1}) - \ln(\overline{Q}_i)$
- risky discount factor $\overline{B}_i = \overline{P}_i \overline{Q}_i$
The Protection Leg

From previous part, for protection leg we have:

\[
P_{PV_{Protection\ Leg}} = -\frac{N(1 - RR)}{P(t_v)} \int_0^T P(s) dQ(s)
\]

\[
= \frac{N(1 - RR)}{P(t_v)} \sum_{i=1}^n I_i \quad \text{where} \quad I_i = -\int_{t_{i-1}}^{t_i} P(s) dQ(s)
\]

The individual integral elements are given by:

\[
I_i = -\int_{t_{i-1}}^{t_i} P(t) dQ(t) = \bar{P}_{i-1} \bar{Q}_{i-1} h_i \int_{t_{i-1}}^{t_i} e^{-(f_i + h_i)(t-t_{i-1})} dt
\]

\[
= \frac{h_i}{f_i + h_i} (\bar{B}_{i-1} - \bar{B}_i) = \frac{\hat{h}_i}{\hat{f}_i + \hat{h}_i} (\bar{B}_{i-1} - \bar{B}_i)
\]
The Premium Leg

We have same simplification for the premium leg:

\[ PV_{\text{premium}} = \frac{NC}{P(t_v)} \sum_{i=1}^{M} \left[ \triangle_i P(t_i)Q(e_i) - \eta_i \int_{s_i}^{e_i} (t - s_i) P(t) \frac{dQ(t)}{dt} dt \right] \]

\[ = \frac{NC}{P(t_v)} \sum_{i=1}^{M} \left[ \triangle_i P(t_i)Q(e_i) + \eta_i I_i \right] \quad \text{where} \quad I_i = - \int_{s_i}^{e_i} (t - s_i) P(t) \frac{dQ(t)}{dt} dt \]

(30)

Since the time interval \((s_i, e_i)\) could possibly contains at least one or more nodes (in yield curve and credit curve). If we now truncate our set of combined nodes, so it only contains the \(n_k\) nodes between \(s_k\) and \(e_k\) exclusively, then add \(s_k\) and \(e_k\) as the first and last node (i.e. \(t_0 = s_k\) and \(t_{n_k} = e_k\)) we have

\[ I_k = - \sum_{i=1}^{n_k} \int_{t_{i-1}}^{t_i} (t - s_k) P(t) \frac{dQ(t)}{dt} dt \]

\[ = \sum_{i=1}^{n_k} \left[ \frac{\hat{h}_i}{\hat{f}_i + \hat{h}_i} \left( \Delta t_{i-1} \left( \frac{\overline{B}_{i-1} - \overline{B}_i}{\hat{f}_i + \hat{h}_i} - \overline{B}_i \right) + (t_{i-1} - s_k)(\overline{B}_{i-1} - \overline{B}_i) \right) \right] \]

(31)
The latest ISDA Source Code’s (version 1.8.2) implementation for $I_k$ gives the formula as:

$$I_{ISDA}^k = \sum_{i=1}^{n_k} \left( \frac{\hat{h}_i}{\hat{f}_i + \hat{h}_i} \left( \Delta t_{i-1} \left( \frac{\hat{B}_{i-1} - \hat{B}_i}{\hat{f}_i + \hat{h}_i} - \hat{B}_i \right) + (t_{i-1} - \tilde{s}_k) (\hat{B}_{i-1} - \hat{B}_i) \right) \right)$$

(32)

where $\tilde{s}_k = s_k - 1/730$, the accrual start is reduced by half a day (on a 365-based day count), the reason for this is unclear. This is simply:

$$I_{ISDA}^k = I_{OpenGamma}^k + \frac{1}{730} \sum_{i=1}^{n_k} \frac{\hat{h}_i}{\hat{f}_i + \hat{h}_i} (\hat{B}_{i-1} - \hat{B}_i)$$

(33)

In a note in December 2012, Markit states that the formula (32) is incorrect. The corrected equation is (in our notation)

$$I_{Markit \ flx}^k = \sum_{i=1}^{n_k} \left[ \frac{\Delta t_{i-1} \hat{h}_i}{\hat{f}_i + \hat{h}_i} \left( \frac{\hat{B}_{i-1} - \hat{B}_i}{\hat{f}_i + \hat{h}_i} - \hat{B}_i \right) \right]$$

(34)

The above equation can be rewrite as:

$$I_{Markit \ flx}^k = I_{OpenGamma}^k - \sum_{i=1}^{n_k} \left[ \frac{\hat{h}_i}{\hat{f}_i + \hat{h}_i} \left( (t_{i-1} - \tilde{s}_k) (\hat{B}_{i-1} - \hat{B}_i) \right) \right]$$

(35)
Formula given in ISDA Standard CDS Model C Code

The figure shows a comparison between ISDA and Markit models. The x-axis represents \( \lambda + F \), and the y-axis represents the value of function A. The blue line indicates the ISDA model, while the red line represents the Markit model. The graph displays similar trends but with slight differences in the values at various points.
Both Markit and Bloomberg still use the code with equation (32) and are yet to switch to equation (34). (Source: OpenGamma Research Paper in Oct 3rd, 2014)

Nevertheless some groups have started using the Markit fix. (Source: OpenGamma Research Paper in Oct 3rd, 2014)
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For a particular reference entity, we consider following two cases:

- There is only one CDS tenor available with reliable pricing information (i.e. is liquid)
- CDS quotes are available for several tenors on the same name, and one is interested in building a credit curve that will exactly reprice all of the quoted instruments.

For market observed clean price, $PV_{\text{clean}}$, define the price residual function:

$$G(\lambda) = \frac{(PV_{\text{Protection Leg}}(\lambda) - C \times RPV01(\lambda) - PV_{\text{clean}})/N}{\lambda}$$  \hspace{1cm} (36)
From the above figure, $G(\lambda)$ is monotonic in $\lambda$ and is guaranteed to have a root $G(\lambda^*) = 0$ provided that

$$-CN \sum_{i=1}^{M} \Delta_i P(t_i) \leq P(t_v)PV_{clean} < (1 - RR)N$$

The above inequality is also the no arbitrage condition for $PV_{clean}$. 

**Figure 1:** The normalised price (i.e. unit notional) for a standard 5Y CDS against hazard rate for a range of recovery rates. The CDS premium is 500bps.
If CDS quotes are available for several tenors on the same name, the problem is then the curve construction problem. Based on the interpolation scheme and a set of nodes, we work with the zero hazard rate curve $\Lambda(t)$. Let $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the vector of curve nodes and $V = (V_1, ..., V_n)^T$ be the vector of market prices, then we must solve the vector equation:

$$G(\Lambda) = V \quad (38)$$

where $G(\cdot)$ is the function that prices all the CDSs given the credit curve that results from the nodes $\Lambda$. We need in effect to invert this equation, and have

$$\Lambda = G^{-1}(V) \quad (39)$$
Multiple CDSs

This can be handled by a multi-dimensional root finding technique. Since the interpolator used in the ISDA model is linear in the log of the survival probability, the survival probability (or equivalently the zero hazard rate) at a particular time only depends on the value of the two adjacent nodes. Thus the vector equation may be broken down as:

\[ G_1(\Lambda_1) = V_1 \]
\[ G_2(\Lambda_1, \Lambda_2) = V_2 \]
\[ \vdots \]
\[ G_n(\Lambda_1, \Lambda_2, \ldots, \Lambda_n) = V_n \]  \hspace{1cm} (40)

We can iteratively solve the equation from the first one to the last one.
<table>
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<tr>
<th>Term</th>
<th>Ticker</th>
<th>Contributor</th>
<th>Bid (bps)</th>
<th>Ask</th>
<th>Update Time</th>
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<td>2) 6 mo</td>
<td>CT679414</td>
<td>CMAN</td>
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<td>6.420</td>
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<td>CMAN</td>
<td>0.680</td>
<td>10.040</td>
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<td>4) 2 yr</td>
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<td>4.000</td>
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<td>8) 7 yr</td>
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<td>60.580</td>
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Thank you!
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3 Appendix